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A New Radiative Equilibrium Model for Investigating Atmospheric Radiation Entropy Flux

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Abstract

A new gray-atmosphere radiative equilibrium model is built to investigate the profile of vertical atmospheric radiation entropy flux. The new model integrates the most accurate expressions available for calculating the Earth's radiation entropy fluxes, and provides analytic solutions for the vertical profiles of temperature, radiation energy flux and radiation entropy flux. It is found that both atmospheric shortwave and net longwave radiation entropy fluxes increase with height, and the latter is about one order larger in magnitude than the former. The profile of the net atmospheric radiation entropy flux follows approximately that of the net longwave radiation entropy flux. Sensitivity study further reveals that a "darker" atmosphere with a larger atmospheric longwave optical depth exhibits a smaller net radiation entropy flux at all altitudes. Furthermore, the magnitude of the decrease of the net radiation entropy flux due to the increase of atmospheric longwave optical depth is almost the same at all altitudes, suggesting an intrinsic

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connection between the net atmospheric radiation entropy flux and atmospheric longwave optical depth. These results indicate that the overall strength of irreversible atmospheric processes as determined by the total atmospheric radiation entropy flux is closely related to the amount of greenhouse gases in the atmosphere.

Keywords: radiative equilibrium model, atmospheric radiation entropy flux, atmospheric optical depth.

1. Introduction

The principles of energy, momentum and mass balances have been the fundamental bases for building the theories of Earth's climate and climate change, ranging from simple energy balance models to state-of-art global climate models. Despite the tremendous progress in our understanding of the climate system resulting from improved climate models, many questions remain unresolved regarding the Earth's climate such as climate forcings and feedbacks (e.g., IPCC 2007; also see http://globalclimatechange.jpl.nasa.gov/uncertainties/). Additional constraint(s) seems necessary to further our theoretical understanding and quantification of the Earth' climate.

The Earth system absorbs solar (shortwave, SW hereafter) radiation energy and emits the same amount of energy in the form of longwave (LW hereafter) radiation. However, the outgoing LW radiation carries much higher entropy than the incoming SW radiation (e.g., Stephens and O'Brien 1993; Wu and Liu 2009). From the perspective of a complex system, the net negative radiation entropy flux drives the overall processes of the Earth system to internally generate entropy to maintain its orderliness. Therefore, it appears natural and necessary to integrate the second law of thermodynamics (e.g., Planck 1922; Prigogine 1980; Rubi 2008) into the theories of the complex Earth's climate system (e.g., Dewar 2003; Kleidon and Lorenz 2005; Whitfield 2005).

Application of the second law of thermodynamics, especially of entropy-related extremal principles, to the study of the Earth's climate has been explored since the 1970s (e.g., Paltridge 1975, 1978; Golitsyn and Mokhov 1978; Nicolis and Nicolis 1980; Grassl 1981; Mobbs 1982; Essex 1984; Peixoco *et al.* 1991; Stephens and O'Brien 1993; Goody 2000; Ozawa *et al.* 2003; Pujol 2003; Paltridge *et al.* 2007; Pauluis 2008). However, theoretical development along this path is still in an infant stage. One central question lies in the role of radiation entropy in shaping the Earth's climate and how to accurately calculate radiation entropy of the Earth system (e.g., Essex 1984; Callies and Herbert 1988; Lesins 1990; Peixoto *et al.* 1991; Stephens and O'Brien 1993; Pelkowski 1994; Goody and Abdou 1996; Goody 2000; Ozawa et al. 2003). We have recently reviewed the existing expressions for estimating radiation entropy flux, constructed several expressions for calculating the Earth's radiation entropy flux based on them, and clarified the applicabilities of those existing expressions for calculating the Earth's radiation entropy flux (Wu and Liu 2009).

Moreover, previous studies have been mainly concerned the Earth's radiation entropy flux when the Earth system – hence its climate – is treated as a whole. The detailed profile of vertical atmospheric radiation entropy flux and its relationship with atmospheric physical properties (such as atmospheric optical depth or total cloud cover) have rarely been investigated. Li *et al.* (1994) investigated the profiles of vertical atmospheric entropy fluxes by using a Canadian Climate Center one-dimensional (1D) radiative convective model. The LW radiation entropy flux in Li *et al.* (1994) was calculated by assuming the LW radiation entropy transfer equation was linear to the LW radiation energy transfer equation [Eqs. (15) and (18) in Li *et al.* 1994]. However, as discussed in Li *et al.* (1994, section 2) and some other papers (Wright *et al.* 2001; Zhang and Basu 2007; Wu and Liu 2009), this linear relationship holds only for blackbody radiation, which is not the case for the Earth's climate (Wu and Liu 2009). Pelkowski (1994) investigated atmospheric LW radiation entropy flux at the top of the atmosphere (TOA) and at the surface for different atmospheric LW optical depths. He first used a radiative energy transfer equation to calculate atmospheric LW radiation energy flux under the assumption of a blackbody Earth's surface absorbing all incident solar radiation (i.e., the divergence of radiative flux within the atmosphere only involves LW radiation; see his section 3). Then he used a radiative entropy transfer equation (linear to radiative energy transfer equation, described in his section 3.1) to calculate atmospheric LW radiation entropy flux. The vertical profile of atmospheric temperature must be given beforehand in his calculation, and four different kinds of vertical temperature profiles were used, including one from a radiative equilibrium model. However, no discussion was given on the detailed profile of vertical atmospheric radiation entropy flux.

In view of the importance of vertical atmospheric radiation entropy flux to constraining the overall energetics of atmospheric processes, here we utilize the new understanding of graybody radiation entropy to extend these rare studies on the profiles of vertical atmospheric radiation entropy flux under different atmospheric conditions. The primary objective of this paper is to build a new radiative equilibrium model that permits the calculation of the vertical profile of the atmospheric radiation entropy flux in addition to the profiles of atmospheric temperature and radiation energy flux. The basic theoretical framework of the new model is described in section 2. Section 3 shows and discusses the results derived from the model. Concluding remarks are followed in section 4.

2. Theoretical framework

The new model comprises two components, which are used to calculate the vertical profiles of temperature and radiation energy flux, and the vertical profile of radiation entropy flux. The two components are described in Section 2.1 and 2.2., respectively.

2.1 Radiation Energy Fluxes

The simplest climate model for analytically solving the vertical profiles (temperature and radiation energy flux) of an idealized steady-state atmosphere is a radiative equilibrium model. This model does not consider vertical or horizontal heat transfer by conduction, air motions or latent heat release and so at any altitude the total radiation energy flux is equal to zero. Atmospheric radiative energy transfer as a key problem involved has been studied extensively and the so-called radiative transfer equation often serves as a governing equation for atmospheric LW radiative energy transfer (e.g., Goody and Yung 1989; Lenoble 1993; Liou 2002). Here we use the radiative transfer equation given in Liou [2002, Eq. (7.4.1)] and Eddington's approximation to build our new radiative equilibrium model. Briefly, the governing equations for the atmospheric LW radiation energy flux in a gray atmosphere is written as

$$\frac{1}{3}\frac{dI_1(\tau)}{d\tau} = I_0(\tau) - \varepsilon B(\tau) \tag{1}$$

$$\frac{dI_0(\tau)}{d\tau} = I_1(\tau) \tag{2}$$

where $\varepsilon B(\tau)$ represents the source (gray atmospheric emission) function [$B(\tau) = \frac{\sigma T^4(\tau)}{\pi}$, σ is Stefan-Boltzmann constant 5.670x10^{-8} W m^{-2} K^{-4}, $T(\tau)$ is the emissive temperature of the gray atmosphere]; τ is the atmospheric LW optical depth at any altitude; $I_0(\tau)$ and $I_1(\tau)$ are two basic components that determine the upward and downward LW radiation energy fluxes coming from Eddington's approximation. The Greek symbol ε represents the effective atmospheric emissivity. It should be emphasized that as will be shown later, the introduction of ε eliminates the problem of surface thermal discontinuity that has long bothered previous 1D vertical climate models (e.g., Pujol and Fort 2002), and separates the new model from existing similar models in this aspect.

The upward [$F_L^{\uparrow}(\tau)$] and downward [$F_L^{\downarrow}(\tau)$] LW radiation energy fluxes can be derived from Eddington's approximation through integration over upward and downward solid angles respectively such that

$$F^{\uparrow}(\tau) = \pi I_0(\tau) + \frac{2\pi}{3} I_1(\tau) \tag{3}$$

$$F^{\downarrow}(\tau) = \pi I_0(\tau) - \frac{2\pi}{3} I_1(\tau) \tag{4}$$

Note that the upward integration spans the azimuth angle from 0 to 2π and the zenith angle from 0 to $\frac{\pi}{2}$ while the azimuth angle is from 0 to 2π and the zenith angle from π to $\frac{\pi}{2}$ for downward integration. The atmospheric SW radiation energy flux is assumed downward only, and obeys Beer's extinction law such that

$$F_S(\tau_S) = F_S(0)\exp(-\tau_S) \tag{5}$$

where $F_S(0)$ represents the SW radiation energy flux at TOA and τ_S represents atmospheric SW optical depth at any altitude. Following Goody and Yung (1989) or Ozawa and Ohmura (1997), we assume a linear relationship between atmospheric SW optical depth τ_S and atmospheric LW optical depth τ, i.e., $\tau_S = a_0\tau$. Therefore, Eq. (5) can be written as

$$F_S(\tau) = F_S(0)\exp(-a_0\tau) \tag{6}$$

At each altitude in the atmospheric column, SW radiation energy flux is balanced with net LW radiation energy flux according to energy conservation law, that is,

$$F_S(\tau) = F_L(\tau) = F_L^{\uparrow}(\tau) - F_L^{\downarrow}(\tau) \tag{7}$$

Substitution of Eqs. (3), (4) and (6) into Eq. (7) yields

$$F_S(0)\exp(-a_0\tau) = \frac{4\pi}{3} I_1(\tau) \tag{8}$$

The TOA boundary condition used is zero downward LW radiation energy flux (i.e., TOA incoming SW radiation energy flux has to be balanced with TOA outgoing LW radiation energy flux). Mathematically, this boundary condition can be written as

$$\frac{Q_0}{4}(1-\alpha_P) = F_S(0) = F_L^{\uparrow}(0) = \pi I_0(0) + \frac{2\pi}{3} I_1(0) \tag{9}$$

where Q_0 is the solar constant and α_P is the planetary albedo.

The set of Eqs. (1), (2) and (8) with the boundary condition (9) can be analytically solved and the solutions are

$$I_0(\tau) = \frac{F_S(0)}{\pi}\left(-\frac{3}{4a_0}\exp(-a_0\tau) + \frac{3}{4a_0} + \frac{1}{2}\right) \tag{10}$$

$$I_1(\tau) = \frac{3F_S(0)}{4\pi}\exp(-a_0\tau) \tag{11}$$

$$T^4(\tau) = \frac{\pi B(\tau)}{\sigma} = \frac{F_S(0)}{\varepsilon\sigma}\left(-\frac{3}{4a_0}\exp(-a_0\tau) + \frac{a_0}{4}\exp(-a_0\tau) + \frac{3}{4a_0} + \frac{1}{2}\right) \tag{12}$$

The effective emissivity ε in the above solutions can be further related to the surface boundary condition that forces the surface air temperature $T(\tau_*)$ equals the ground temperature T_g (τ_* is the overall atmospheric LW optical depth). Mathematically, this surface boundary condition can be written as

$$T_g^4 = T^4(\tau_*) = \frac{F_S(0)}{\varepsilon\sigma}\left(-\frac{3}{4a_0}\exp(-a_0\tau_*) + \frac{a_0}{4}\exp(-a_0\tau_*) + \frac{3}{4a_0} + \frac{1}{2}\right) \tag{13}$$

or

$$\varepsilon = \frac{F_S(0)}{\sigma T_g^4}\left(-\frac{3}{4a_0}\exp(-a_0\tau_*) + \frac{a_0}{4}\exp(-a_0\tau_*) + \frac{3}{4a_0} + \frac{1}{2}\right) \tag{13'}$$

where a_0 can be determined by $a_0 = \tau_S^* / \tau_*$ when the overall atmospheric SW (τ_S^*) and LW (τ_*) optical depths are both given.

The ground temperature T_g can be determined from the ground energy balance equation, i.e., the ground emissive energy flux $\varepsilon_g \sigma T_g^4$ is balanced with the ground absorbed SW radiation energy flux $F_S(\tau_*)$ and LW radiation energy flux $F_L^{\downarrow}(\tau_*)$ emitted by surface air,

$$\varepsilon_g \sigma T_g^4 = F_S(\tau_*) + F_L^{\downarrow}(\tau_*) \tag{14}$$

or

$$T_g^4 = \frac{F_S(\tau_*) + F_L^{\downarrow}(\tau_*)}{\varepsilon_g \sigma} \tag{14'}$$

where ε_g is the ground emissivity ($\varepsilon_g = 1.0$ if the ground is assumed acting as a blackbody as in this study). The value of the ground absorbed SW radiation energy flux $F_S(\tau_*)$ can be obtained directly from Eq. (6). The value of the ground absorbed LW radiation energy flux $F_L^{\downarrow}(\tau_*)$ can be calculated by substituting the expressions of $I_0(\tau)$ [Eq. (10)] and $I_1(\tau)$ [Eq. (11)] into Eq. (4).

One advantage of this new model compared with other 1D vertical climate models is that it eliminates the surface thermal discontinuity problem that has long bothered other 1D vertical climate models. In addition, for any given atmospheric LW optical depth, atmospheric temperature, radiation energy fluxes and the corresponding effective atmospheric emissivity can be readily obtained in simple analytic forms.

2.2 Radiation Entropy Fluxes

The calculation of radiation entropy flux has attracted attention of many different fields (such as Engineering or Earth Science) for several decades (e.g., Petela 1964; Noda and Tokioka

1983; Peixoco et al. 1991; Stephens and O'Brien 1993; Wright et al. 2001; Petela 2003; Ozawa et al. 2003; Zhang and Basu 2007). Several existing expressions were proposed to calculate radiation entropy flux in previous studies, and the results (such as the Earth's radiation entropy flux) from using different expressions are sometimes far from each other (Wu and Liu 2009). As mentioned above, Wu and Liu (2009) have recently examined the applicability of the various expressions for calculating the Earth's radiation entropy flux, where some new expressions were constructed for calculating the Earth's SW or LW radiation entropy fluxes based on them. It is desirable to couple the expressions, which show the best performance in calculating the Earth's SW or LW radiation entropy fluxes, with the new radiative equilibrium model to calculate the vertical profiles of atmospheric radiation entropy fluxes. In this paper, the expressions for calculating vertical atmospheric SW and LW radiation entropy fluxes are constructed based on the expressions by Stephens and O'Brien (1993) and by Wright *et al.* (2001) respectively, considering that the two expressions show the best ability in estimating the Earth's SW or LW radiation entropy fluxes respectively according to Wu and Liu (2009). Discussions on the details of the *pros* and *cons* of the two expressions can be found in Wu and Liu (2009).

Suppose that I_ν^{Sun} is the spectral energy flux of TOA incident solar (SW) radiation per solid angle per frequency (W m^{-2} sr^{-1} s) and $\delta(\tau) I_\nu^{Sun} = I_\nu^{Sun}(\tau)$ is the SW spectral energy flux at any altitude, the SW radiation entropy flux $J_S(\tau)$ can be written as (Stephens and O'Brien 1993)

$$J_S(\tau) = \int_0^\infty \frac{2\pi\kappa\nu^2}{c^2} \left\{ \left(1 + \frac{c^2\delta(\tau)I_\nu^{Sun}}{2h\nu^3}\right) \ln\left(1 + \frac{c^2\delta(\tau)I_\nu^{Sun}}{2h\nu^3}\right) - \frac{c^2\delta(\tau)I_\nu^{Sun}}{2h\nu^3} \ln \frac{c^2\delta(\tau)I_\nu^{Sun}}{2h\nu^3} \right\} d\nu$$

$$= \frac{4}{3}\sigma T_{Sun}^3 \delta(\tau)\left[0.96515744 - 0.27765652 \ln \delta(\tau)\right] \quad (15)$$

where T_{Sun} is the Sun's emissive temperature 5779 K, h, c, κ and ν are Planck's constant 6.626x10^{-34} J s, speed of light in vacuum 2.9979x10^{8} m s^{-1}, Boltzmann constant 1.381x10^{-23} J K^{-1} and frequency respectively. Note that, here we remain the same assumption as Stephens and O'Brien (1993) that the Sun illuminates a Lambertian spherical-geometrical surface of the Earth system, that is, solar radiation energy flux is the same in all direction for the Earth system and independent of the direction of incident solar radiation. Thus the ratio $\delta(\tau)$ of each-altitude SW spectral energy flux to that at TOA is equal to $\frac{4\cos\theta_0\Omega_0}{Q_0\pi}F_S(\tau)$, where cosine solar zenith angle to the Earth $\cos\theta_0$ is set 0.25 and the solar solid angle to the Earth Ω_0 is 67.7 x 10^{-6} sr.

The net LW radiation entropy flux at each altitude $J_L(\tau)$ is calculated through its effective upward and downward LW radiation entropy fluxes, i.e.,

$$J_L(\tau) = J_L^{\uparrow}(\tau) - J_L^{\downarrow}(\tau) \tag{16}$$

For simplicity, we assume that the upward [$F_L^{\uparrow}(\tau)$] and downward [$F_L^{\downarrow}(\tau)$] LW radiation energy fluxes are equivalent to the same energy fluxes from gray atmospheric emission. So, the upward [$I_\nu^{\uparrow}(\tau)$] and downward [$I_\nu^{\downarrow}(\tau)$] LW spectral radiation energy fluxes can be uniquely determined through

$$F_L^{\uparrow}(\tau) = \int d\nu \int_{\Omega^{\uparrow}} I_\nu^{\uparrow}(\tau)\cos\theta d\Omega = \varepsilon\sigma[T_L^{\uparrow}(\tau)]^4 \tag{17}$$

$$F_L^{\downarrow}(\tau) = \int d\nu \int_{\Omega^{\downarrow}} I_\nu^{\downarrow}(\tau)\cos\theta d\Omega = \varepsilon\sigma[T_L^{\downarrow}(\tau)]^4 \tag{18}$$

where $T_L^{\uparrow}(\tau)$ and $T_L^{\downarrow}(\tau)$ are equivalent emissive temperatures of the upward and downward LW radiation energy fluxes, $\Omega^{\uparrow}$ and $\Omega^{\downarrow}$ represent the ranges of upward and downward solid angles (upward $\Omega^{\uparrow}$: azimuth angle from 0 to 2π and zenith angle from 0 to $\frac{\pi}{2}$; downward $\Omega^{\downarrow}$: azimuth angle from 0 to 2π and zenith angle from π to $\frac{\pi}{2}$). According to Wright *et al.* (2001), the upward [$J_L^{\uparrow}(\tau)$] and downward [$J_L^{\downarrow}(\tau)$] LW radiation entropy fluxes can be calculated as

$$J_L^{\uparrow}(\tau) = \int_0^{\infty} \frac{2\pi\kappa\nu^2}{c^2} \left\{ \left(1 + \frac{c^2 I_\nu^{\uparrow}(\tau)}{2h\nu^3}\right) \ln\left(1 + \frac{c^2 I_\nu^{\uparrow}(\tau)}{2h\nu^3}\right) - \frac{c^2 I_\nu^{\uparrow}(\tau)}{2h\nu^3} \ln \frac{c^2 I_\nu^{\uparrow}(\tau)}{2h\nu^3} \right\} d\nu$$

$$= \frac{15\sigma}{\pi^4} \varepsilon[\frac{4\pi^4}{45} - (c2 - c3\varepsilon)\log\varepsilon][T_L^{\uparrow}(\tau)]^3 \tag{19}$$

$$J_L^{\downarrow}(\tau) = \int_0^{\infty} \frac{2\pi\kappa\nu^2}{c^2} \left\{ \left(1 + \frac{c^2 I_\nu^{\downarrow}(\tau)}{2h\nu^3}\right) \ln\left(1 + \frac{c^2 I_\nu^{\downarrow}(\tau)}{2h\nu^3}\right) - \frac{c^2 I_\nu^{\downarrow}(\tau)}{2h\nu^3} \ln \frac{c^2 I_\nu^{\downarrow}(\tau)}{2h\nu^3} \right\} d\nu$$

$$= \frac{15\sigma}{\pi^4} \varepsilon[\frac{4\pi^4}{45} - (c2 - c3\varepsilon)\log\varepsilon][T_L^{\downarrow}(\tau)]^3 \tag{20}$$

where parameters c_2 and c_3 are 2.336 and 0.260 respectively (Wu and Liu 2009).

3. Results

It is clear that the vertical profiles of atmospheric structures (temperature, radiation energy flux and radiation entropy flux) can be evaluated through the equations derived in section 2. The following expression is used to convert the atmospheric LW optical depth into atmospheric height z,

$$\tau(z) = \tau_* \exp(-\frac{z}{H_W}) \quad (21)$$

where H_W represents a typical scale height of atmospheric water vapor, approximately 2000 m. Note that this expression, which assumes that τ is mainly affected by atmospheric water vapor, has been widely used for modeling the Earth's climate (e.g., Goody and Yung 1989; Pelkowski 1994; Ozawa and Ohmura 1997; Pujol and Fort 2002). The solar constant Q_0 = 1367 W m^{-2} and the planetary albedo α_P = 0.30 are used in the calculation. The overall atmospheric SW optical depth τ_S^* = 0.53 is used in the calculation according to the analysis by Ozawa and Ohmura (1997) of the measurements of the SW radiation energy fluxes at the Earth's surface and at TOA.

The fidelity of the new model is first checked by analyzing the simulated vertical structures of the atmospheric temperature and radiation energy flux. Figure 1a shows the simulated vertical temperature profile as the overall atmospheric LW optical depth τ_* equals 2.0, 3.0 or 4.0. Evidently, the atmospheric temperature decreases with height (faster in the lower troposphere and much slower in the upper troposphere). Furthermore, when the atmospheric LW optical depth τ_* increases, the atmospheric temperature increases in the lower troposphere and decreases in the upper troposphere. The effective atmospheric emissivities corresponding to τ_*=2.0, 3.0 or 4.0 are 0.870, 0.894 or 0.912, respectively. It is shown that a larger atmospheric LW optical depth τ_* leads to a larger effective atmospheric emissivity, i.e., a darker atmosphere in terms of the LW radiation. As a consequence from the radiative equilibrium model, a darker atmosphere traps more heat (leading to larger temperature) in lower troposphere (this characteristic was captured also by Ozawa and Ohmura 1997 from a radiative convective model), and traps less heat (leading to smaller temperature) in upper troposphere. However, both upward and downward LW radiation energy fluxes are larger in the whole atmospheric column for a darker atmosphere, especially in the lower troposphere (Figure 1b). These results indicate that despite its simplicity, the new model yields the basic atmospheric structures reasonably well, and more importantly, removes the problem of the surface discontinuity that has long bothered other similar models.

Next, we examine the simulated profile of vertical atmospheric radiation entropy flux. Figure 2a shows the upward and downward LW radiation entropy fluxes. Like the upward and downward LW radiation energy fluxes, both upward and downward LW radiation entropy fluxes decrease with height. The basic structures are qualitatively similar to those reported in Li *et al.* (1994, see their Fig. 1). The magnitude of the upward LW radiation entropy flux at the Earth's surface and the magnitude of the downward LW radiation entropy flux in the whole atmospheric column as τ_* equals 2.0 or 3.0 are also similar to those in Li *et al.* (1994). However, the magnitude (1.25 W m^{-2} K^{-1}, 1.23 W m^{-2} K^{-1} or 1.21 W m^{-2} K^{-1}) of the upward LW radiation entropy flux in the upper troposphere (or at TOA) as τ_* equals 2.0, 3.0 or 4.0 is much smaller than that (~1.75 W m^{-2} K^{-1}) in Li *et al.* (1994) while close to that (1.22 W m^{-2} K^{-1}) in Pelkowski (1994), who used a radiative energy transfer equation for calculating atmospheric LW radiation energy flux and then a radiative entropy transfer equation (linear to his radiative energy transfer equation) for calculating atmospheric LW radiation entropy flux, under the assumption of a blackbody Earth's surface absorbing all incident solar radiation along with the vertical atmospheric temperature profile from a radiative equilibrium model. A darker atmosphere corresponds to larger upward and downward LW radiation entropy fluxes, except for a slight decrease of the upward LW radiation entropy flux in the upper troposphere (or at TOA) [a similar but very slight trend was also obtained from Pelkowski (1994); see the last fourth column in his Table.2]. The downward LW radiation entropy flux shows larger increase than the upward LW radiation entropy flux as the atmosphere increases its darkness (i.e., LW optical depth). In other words, the net atmospheric LW radiation entropy flux decreases as atmosphere LW optical depth increases. This characteristic was partly captured in the results by Pelkowski (1994) [the last second or third columns (net TOA or negative net surface atmospheric LW radiation entropy flux) in his Table 2] although the trend shown in his last second column of Table 2 is very slight. However, no discussion was given in Pelkowski (1994) about whether or not the characteristic is also valid within the whole atmospheric column. It is surprising that the magnitude of the decrease of the net atmospheric LW radiation entropy flux at each altitude keeps almost the same as the atmosphere gets darker (Figure 2b), implying a close linkage between the magnitude of the net atmospheric LW radiation entropy flux and the atmospheric darkness. The value of the net TOA atmospheric LW radiation entropy flux is respectively 1.246 W m^{-2} K^{-1}, 1.221 W m^{-2} K^{-1} or 1.203 W m^{-2} K^{-1} as τ_* equals 2.0, 3.0 or 4.0, slightly larger than the result (1.17 W m^{-2} K^{-1}) in Pelkowski (1994).

The atmospheric SW radiation entropy flux increases with height (Figure 3a) as a direct result of the increase of atmospheric SW radiation energy flux with height. The magnitude of the atmospheric SW radiation entropy flux is about one order smaller than that of the net atmospheric LW radiation entropy flux. The value of the TOA atmospheric SW radiation entropy flux is 0.244 W m^{-2} K^{-1}. As a result, the net radiation entropy flux of the gray atmosphere increases with height (Figure 3b). The darker the atmosphere is (i.e., larger atmospheric LW optical depth), the smaller the net atmospheric radiation entropy flux at all altitudes. The values of the net TOA atmospheric radiation entropy flux are respectively 1.002 W m^{-2} K^{-1}, 0.977 W m^{-2} K^{-1} or 0.959 W m^{-2} K^{-1} as τ_* equals 2.0, 3.0 or 4.0. These results (if subtracting the corresponding net surface atmospheric radiation entropy flux 0.339 W m^{-2} K^{-1}, 0.294 W m^{-2} K^{-1} or 0.263 W m^{-2} K^{-1}) leads to the total atmospheric radiation entropy flux 0.663 W m^{-2} K^{-1}, 0.683 W m^{-2} K^{-1} or 0.696 W m^{-2} K^{-1} (i.e., a measure of overall strength of irreversible atmospheric processes generated within the Earth's atmosphere), because the integration of the atmospheric radiation entropy flux within the atmospheric column equals the net atmospheric radiation entropy flux at TOA minus that at the surface. If we further add the net ground radiation entropy flux produced by ground (blackbody) LW emission and ground SW and LW radiation absorptions [i.e., $\frac{4}{3}\sigma T_g^4 - J_S(\tau_*) - J_L^{\downarrow}(\tau_*)$, 0.339 W m^{-2} K^{-1}, 0.347 W m^{-2} K^{-1} or 0.367 W m^{-2} K^{-1} for τ_* 2.0, 3.0 or 4.0], and the Earth's reflected TOA SW radiation entropy flux 0.032 W m^{-2} K^{-1} (Wu and Liu 2009) into the total atmospheric radiation entropy flux, we obtain the total entropy flux of the Earth system are 1.034 W m^{-2} K^{-1}, 1.062 W m^{-2} K^{-1} or 1.095 W m^{-2} K^{-1} as τ_* equals 2.0, 3.0 or 4.0, slightly smaller than the total Earth's entropy flux 1.272-1.284 W m^{-2} K^{-1} obtained from zero-dimensional climate model by Wu and Liu (2009). Considering that the present results are obtained from such an idealized simple radiative equilibrium model (no convection or other atmospheric processes involved), it is not difficult to understand the estimated slightly smaller value of the total entropy flux of the Earth's system.

4. Conclusions

A new model is presented that allows the evaluation of vertical profiles of atmospheric SW and LW radiation entropy fluxes as well as atmospheric temperature and radiation energy fluxes. Vertical profiles of atmospheric radiation entropy fluxes are investigated. It is found that both atmospheric SW and net LW radiation entropy fluxes increases with height and the latter is about one order larger in magnitude than the former, consistent with the results from other studies (e.g., Li et al. 1994). It is striking that in the radiative equilibrium model, even with the same SW radiation energy deposited (absorbed) at each height, a darker atmosphere with a thicker atmospheric LW optical depth leads to a smaller net radiation entropy flux at all altitudes. The magnitude of the decrease of the net radiation entropy flux because of the increase of atmospheric LW darkness shows almost the same at all altitudes.

It should be emphasized that to the best of our knowledge, the effective atmospheric emissivity is introduced into a radiative equilibrium model for the first time, and this introduction not only eliminates the surface thermal discontinuity problem that has long bothered similar 1D vertical climate models, but also provides a natural link between the radiative equilibrium model and the expressions available for calculating the Earth's radiation entropy fluxes according to Wu and Liu (2009). A simple check for the model's general ability shows that this new model is able to simulate atmospheric temperature and radiation energy flux reasonably well. In addition, the analytic solutions from this new radiative equilibrium model, the vertical profiles of atmospheric temperature and radiation energy fluxes as well as the corresponding effective atmospheric emissivity, can be easily obtained as a function of the atmospheric altitude (LW optical depth or height). Furthermore, the analytic solutions are in simple forms and thus easy to be used for other applications.

It is noteworthy that the results derived from the new model reveal an intrinsic connection between the magnitude of the net atmospheric radiation entropy flux and the overall atmospheric LW optical depth, which further implies that the overall strength of irreversible atmospheric processes as measured by the total entropy production within the Earth's atmosphere is clearly sensitive to increased atmospheric greenhouse gases (i.e., increased atmospheric LW optical depth). Application of this new model in climate change is underway. Also noted is that processes such as clouds have been explicitly neglected in this simple radiative equilibrium model. Future effort will be to generalize this model to consider cloud-related processes such as convection, and examine the role of clouds in determining atmospheric energy and entropy profiles.

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Figure Captions

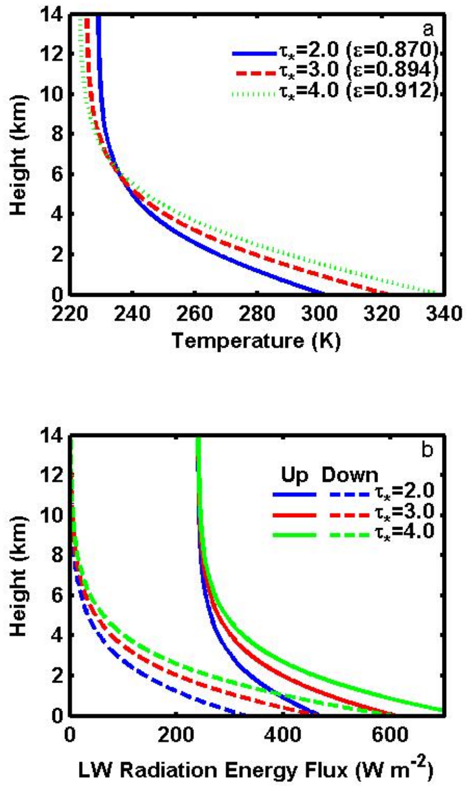
Figure 1. Vertical profiles of atmospheric temperature (a) and LW radiation energy fluxes (b) when the atmospheric LW optical depth equals 2.0, 3.0 or 4.0.

Figure 2. Vertical profiles of upward or downward (a), and net (b) atmospheric LW radiation entropy fluxes when the atmospheric LW optical depth equals 2.0, 3.0 or 4.0.

Figure 3. Vertical profiles of atmospheric SW (a) and net (b) radiation entropy fluxes when the atmospheric LW optical depth equals 2.0, 3.0 or 4.0.

Short title for page headings

Atmospheric radiation entropy flux



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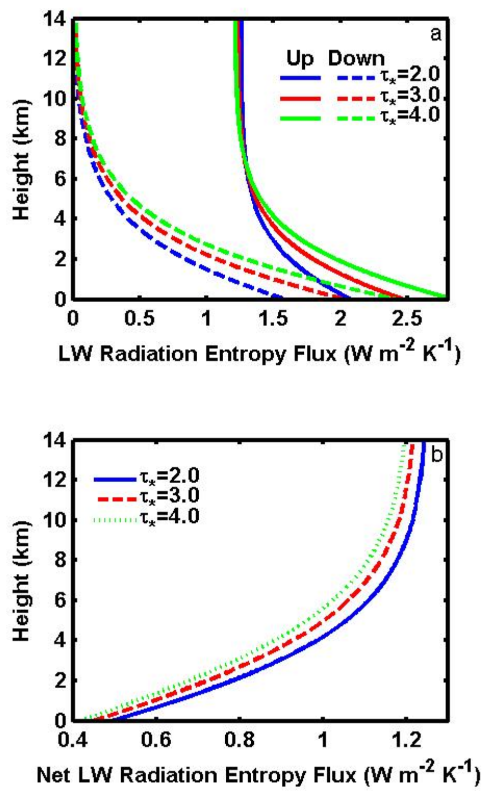

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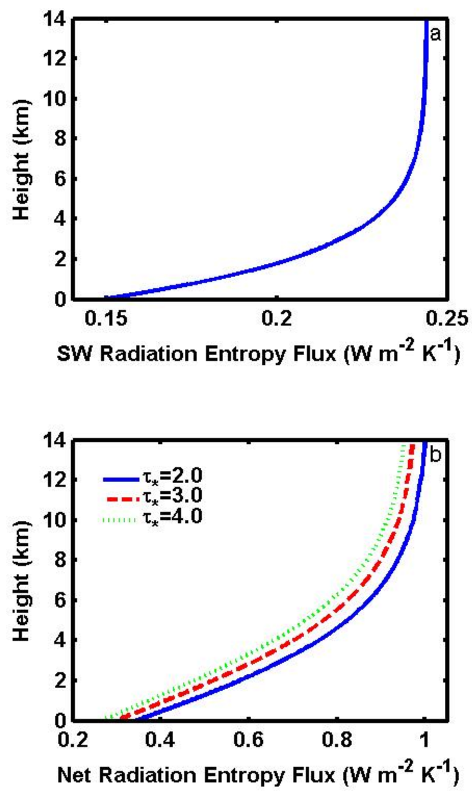

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